RELATIONS AND FUNCTIONS MODULE 1

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REVISION

TOPICS

- EQUIVALENCE RELATION
- TYPES OF FUNCTIONS
- COMPOSITION OF FUNCTIONS & INVERTIBLE FUNCTIONS

TODAY'S TOPIC IS

- **EQUIVALENCE RELATION**

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Reflexive Relation

- A relation **R** on a set **A** is reflexive if **(a,a)** ϵ **R** for all **a** ϵ **A** i.e.
  if **a R a** for all **a** ϵ **A**.
- It means **R** is reflexive if every element of **a** ϵ **A** is related to itself.

For example:

Let **A**={1,2} and let **R**={(1,1),(2,2)}.

Then R is said to be reflexive relation.

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Example of Reflexive Relation

- Consider the following relations on {1, 2, 3, 4}:
- *R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},*
- *R2 = {(1, 1), (1, 2), (2, 1)},*
- *R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},*
- *R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},*
- *R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},*
- *R6 = {(3, 4)}.*
- Which of these relations are reflexive?
- *Solution: The relations R3 and R5 are reflexive because they both contain all pairs of the form (a, a), namely, (1, 1), (2, 2), (3, 3), and (4, 4). The other relations are not reflexive because* they do not contain all of these ordered pairs.
- In particular, *R1, R2, R4, and R6 are not reflexive* because *(3, 3) is not in any of these relations.*

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Symmetric Relation

- A relation **R** on a set **A** is symmetric if whenever **a R b** , then **b R a** for all **a,b** ϵ **A**.
- It means if **a** is related to **b** then **b** also related to **a**.

For example:

Let A={1,2,3} and let

R={(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)}

Then R is symmetric relation as (a,b) ϵ R and (b,a) ϵ R.

Example

Example: Consider the following relations on $\{1, 2, 3, 4\}$

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$

$R_2 = \{(1,1), (1,2), (2,1)\}$ yes

$R_3 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (3,4), (4,1), (4,4)\}$

$R_4 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

$R_5 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$

$R_6 = \{(3,4)\}$

Which of these relations are symmetric ?

TRANSITIVE relations

Definition: A relation R on a set A is **transitive** if whenever aRb and bRc then aRc, that is, if whenever $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$.

Thus R is **not transitive** if there exist a, b, c ∈ R such that $(a,b) \in R$ and $(b, c) \in R$ but $(a,c) \notin R$.

If such a, b and c not exist, then R is transitive.

Example: Consider the following relations on the set $A = \{1, 2, 3\}$:

$$R_1 = \{(1, 1), (1, 2), (2, 3), (1, 3)\}$$
$$R_2 = \{(1, 1), (1, 2), (2,2), (2,3)\}$$
$$R_3 = \{(1, 1), (1, 2), (1,3), (3,3)\}$$

Determine which relation is transitive.

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Equivalence Relations

Definition: A relation R on a set A is called an equi-valence relation if it is reflexive, symmetric and transitive.

Example: Let $A = \{1, 2, 3, 4\}$ and let

$$R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)\}.$$

Then R is an equivalence relation.

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Q1. Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set $A = \{1, 2, 3 \ldots 13, 14\}$ defined as
$R = \{(x, y): 3x - y = 0\}$

<u>Soln</u> : (i) $A = \{1, 2, 3 \ldots 13, 14\}$
$R = \{(x, y): 3x - y = 0\}$
$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$
R is not reflexive since $(1, 1), (2, 2) \ldots (14, 14) \notin R$.
Also, R is not symmetric as $(1, 3) \in R$, but $(3, 1) \notin R$. $[3(3) - 1 \neq 0]$
Also, R is not transitive as $(1, 3), (3, 9) \in R$, but $(1, 9) \notin R$.
$[3(1) - 9 \neq 0]$
Hence, R is neither reflexive, nor symmetric, nor transitive.

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Question 2:

Show that the relation R in the set **R** of real numbers, defined as $R = \{(a, b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

Answer :

$R = \{(a, b): a \leq b^2\}$

It can be observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin \mathbf{R}$, since $\frac{1}{2} > \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

∴R is not reflexive.

Now, $(1, 4) \in R$ as $1 < 4^2$

But, 4 is not less than 1^2.

∴$(4, 1) \notin R$

∴R is not symmetric.

Now,

$(3, 2), (2, 1.5) \in R$

(as $3 < 2^2 = 4$ and $2 < (1.5)^2 = 2.25$)

But, $3 > (1.5)^2 = 2.25$

∴$(3, 1.5) \notin R$

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

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Example 5 Show that the relation R in the set Z of integers given by

$$R = \{(a, b) : 2 \text{ divides } (a - b)\}$$

is an equivalence relation

R is reflexive, as 2 divides $(a - a)$ for all $a \in Z$

if $(a, b) \in R$, then 2 divides $(a - b)$. Therefore, 2 divides $(b - a)$.
Hence, $(b, a) \in R$, which shows that R is symmetric.

if $(a, b) \in R$ and $(b, c) \in R$, then $(a - b)$ and $(b - c)$ are divisible by 2.
Now, $a - c = (a - b) + (b - c)$ is even. So, $(a - c)$ is divisible by 2. This shows that R is transitive. Thus, R is an equivalence relation in Z

Question 8:

Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a-b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

Answer :

$A = \{1, 2, 3, 4, 5\}$

$R = \{(a, b) : |a-b| \text{ is even}\}$

It is clear that for any element $a \in A$, we have $|a-a| = 0$ (which is even).

$\therefore$ R is reflexive.

Let $(a, b) \in R$.

$\Rightarrow |a-b|$ is even.

$\Rightarrow |-(a-b)| = |b-a|$ is also even.

$\Rightarrow (b, a) \in R$

$\therefore$ R is symmetric.

Now, let $(a, b) \in R$ and $(b, c) \in R$.

$\Rightarrow |a-b|$ is even and $|b-c|$ is even.

$\Rightarrow (a-b)$ is even and $(b-c)$ is even.

$\Rightarrow (a-c) = (a-b) + (b-c)$ is even. [Sum of two even integers is even]

$\Rightarrow |a-c|$ is even.

$\therefore$ R is transitive.

Hence, R is an equivalence relation.

Let $E = \{1,3,5\} \subset A$,

then $E \times E = \{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5)\} \subset R$,

Hence Every element of E is related to each other.

Let $F = \{2,4\} \subset A$,

$F \times F = \{(2,2),(2,4),(4,2),(4,4)\} \subset R$

Hence Every element of F is related to each other.

But $E \times F = \{(1,2),(1,4),(3,2),(3,4),(5,2),(5,4)\} \not\subset R$ and

no element of $E \times F$ is an element of R.

Hence no element E is related to any element of F

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HOME WORK

EX 1.1: 3 , 4 , 12 , 14

THANK YOU